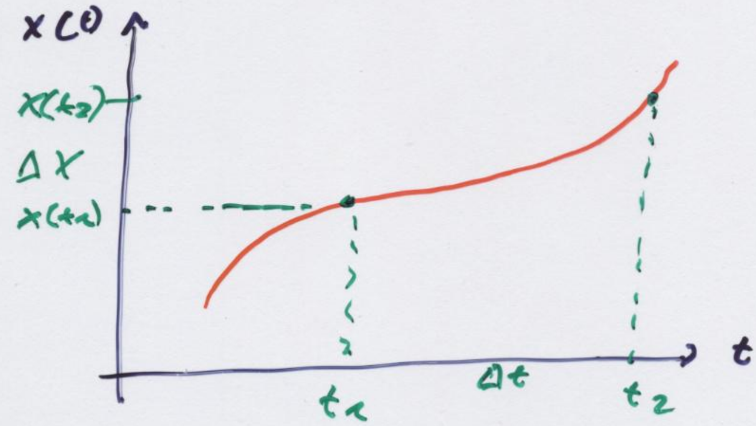
2 Klassische Mechanik

2.1. Mechanik von Massepunkten

2.1.1 Bewegung in 1 Dimension



• Geschwindigkeit

$$\langle v \rangle = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

$$= \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$[v] = \frac{m}{s}$$

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt}$$

$$\hookrightarrow \quad x_2 = x_1 + \int_{t_1}^{t_2} v(t)\, dt$$

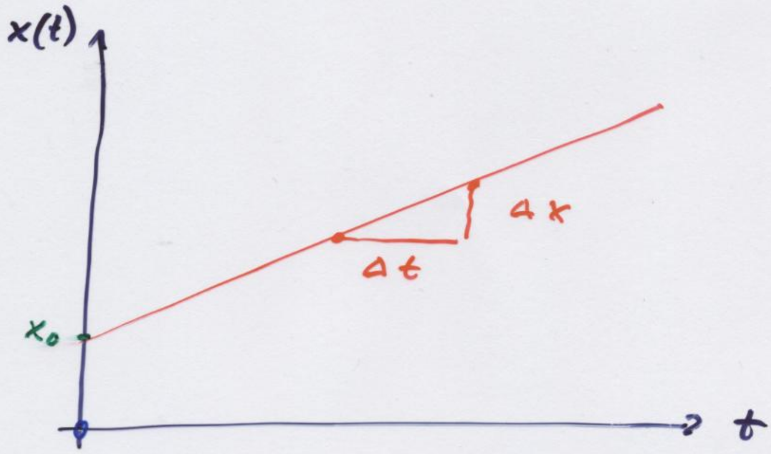
- <u>Beschleunigung</u>

$$\langle a \rangle = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

$$[a] = \frac{m}{s^2}$$

Spezialfall a) : Unbeschleunigte Bewegung



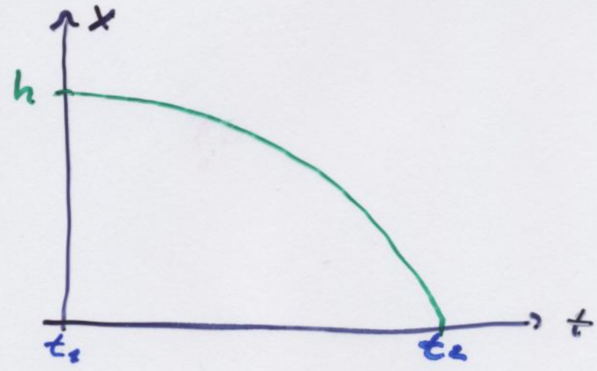
$a(t) = 0$ $\qquad\qquad$ $x(t) = x_0 + v_0 t$

$v(t) = v_0$

b) Gleichförmig beschleunigte Bew.

$$a(t) = a_0$$
$$v(t) = a_0 \cdot t + v_0$$
$$x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0$$

Bsp: Fallender Stein



$a_0 = -g$ Erdbeschleunigung
$v_0 = 0$
$x_0 = h$

$$\Rightarrow x(t) = -\frac{1}{2} g t^2 + h$$
$$v(t) = -gt$$
$$x(t=0) = h$$

$$x(t_2) = 0$$

$$\rightarrow t_2 = \sqrt{2\, h/g}$$

Anwendung: Bestimmung von g (Erdbeschl.)

Bsp. Unfall in der Stadt

Aufprall gegen Wand bei $50 \frac{km}{h}$

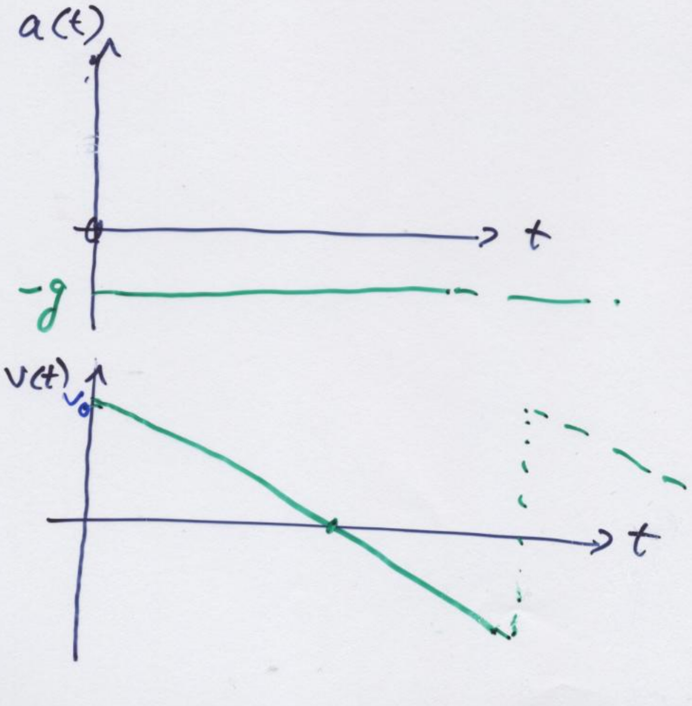
$v = 13{,}9 \frac{m}{s}$

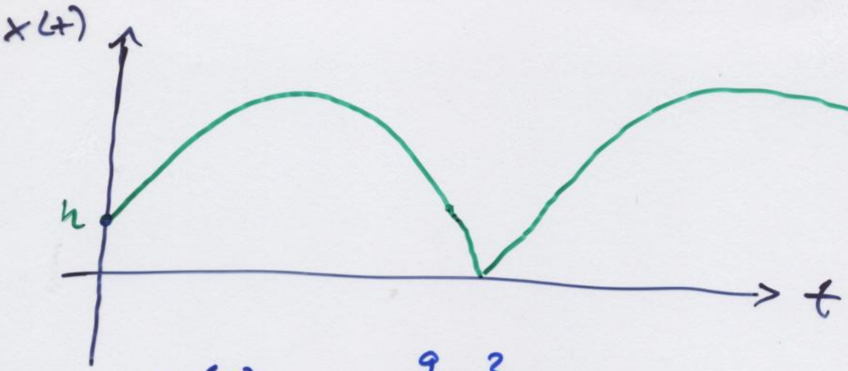
mit $t = \frac{v}{g}$:

$$h = \frac{g}{2} t^2 = \frac{v^2}{2g}$$

$\Rightarrow h = 9{,}9 \text{ m}$

Bsp. Kind spielt Ball





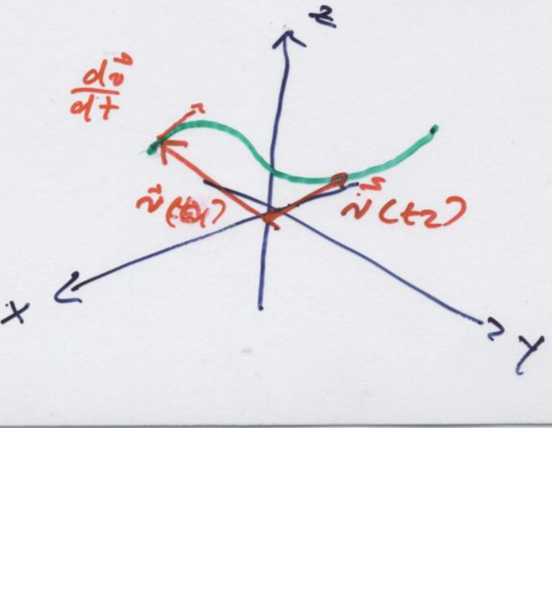
$$x(t) = -\frac{g}{2}t^2 + v_0 t + h$$

2.1.2 Räumliche Bewegung

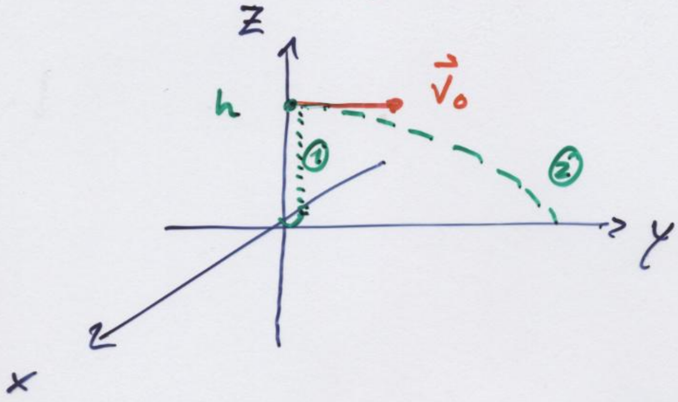
$$\vec{r}\,(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \equiv x(t)\,\vec{e}_x + y(t)\cdot\vec{e}_y + z(t)\,\vec{e}_z$$

$$\vec{v}\,(t) = \begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix} = \begin{pmatrix} dx/dt \\ dy/dt \\ dz/dt \end{pmatrix} \equiv \frac{d\vec{r}}{dt}$$

$$\vec{a}\,(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$



Bs: Fall mit / ohne horizontaler Bewegung



① : $\vec{r}(t) = -\frac{g}{2}t^2\,\vec{e}_z + h\cdot\vec{e}_z$

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{g}{2}t^2 + h \end{pmatrix}$$

Fallzeit: $t_f = \sqrt{\frac{2h}{g}}$; Fallstrecke: $z(t_f) = h$

② :

$$\vec{r}(t) = -\frac{g}{2}t^2\,\vec{e}_z + v_0 t\,\vec{e}_y + h\cdot\vec{e}_z$$

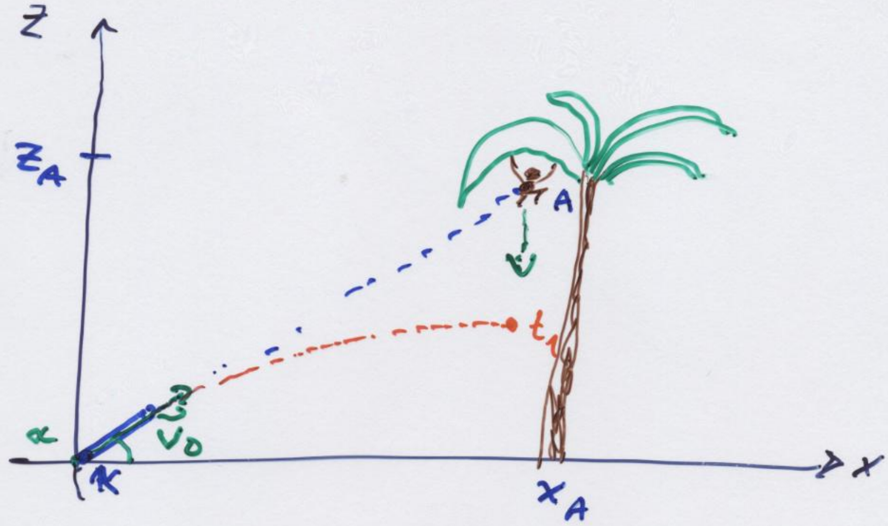
$$= \begin{pmatrix} 0 \\ v_0 t \\ -g/2\, t^2 + h \end{pmatrix}$$

Fallzeit: $z(t_f) = -\frac{g}{2}t_f^2 + h = 0$

$$\Rightarrow t_f = \sqrt{\frac{2h}{g}}$$

Fallstrecke: $y(t_f) = v_0\cdot t_f = v_0\cdot\sqrt{\frac{2h}{g}}$

Bs: Affe im Baum



$$\vec{r}_A(t=0) = \begin{pmatrix} x_A \\ z_A \end{pmatrix} \; ; \quad \vec{r}_A(t) = \vec{r}_A - \frac{g}{2} t^2 \vec{e}_z$$

$$\vec{r}_K(t=0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \; ; \quad \vec{r}_K(t) = \vec{v}_0 t - \frac{g}{2} t^2 \vec{e}_z$$

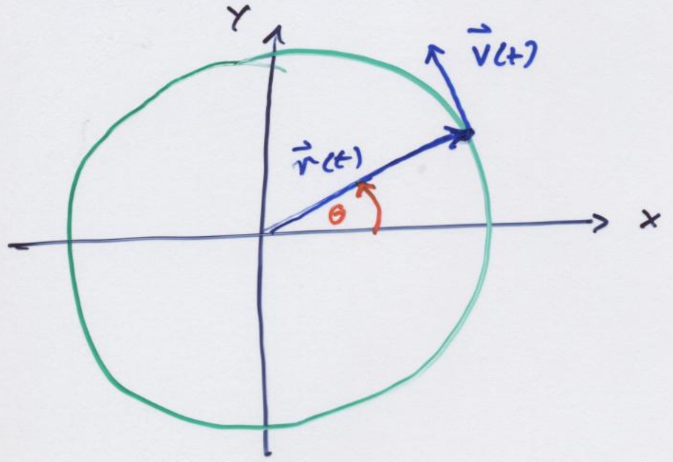
$$\vec{r}_K(t_1) \overset{!}{=} \vec{r}_A(t_1)$$

Kugel trifft

$$\begin{pmatrix} v_{0x} t_1 \\ v_{0z} t_1 - \frac{g}{2} t_1^2 \end{pmatrix} \overset{!}{=} \begin{pmatrix} x_A \\ z_A - \frac{g}{2} t_1^2 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} x_A &= v_{0x} t_1 \\ z_A &= v_{0z} t_1 \end{aligned}\right\} \Rightarrow \frac{z_A}{x_A} = \frac{v_{0z}}{v_{0x}} = \tan\alpha$$

2.1.3 Sonderfall Kreisbewegung



a). Generell: $\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

$$= r(t) \cdot \begin{pmatrix} \cos\theta(t) \\ \sin\theta(t) \end{pmatrix}$$

b) Kreis: $|\vec{r}(t)| = r = \text{const} > 0$

$$\vec{r}(t) = r \cdot \begin{pmatrix} \cos\theta(t) \\ \sin\theta(t) \end{pmatrix}$$

$$\equiv r \cdot \vec{e}_r(t)$$

$$\vec{v}(t) = r \cdot \begin{pmatrix} d/dt \, \cos\theta(t) \\ d/dt \, \sin\theta(t) \end{pmatrix}$$

$$= r \cdot \frac{d\theta}{dt} \begin{pmatrix} -\sin\theta(t) \\ \cos\theta(t) \end{pmatrix}$$

$$= r \cdot \frac{d\theta}{dt} \begin{pmatrix} \cos(\theta(t) + \frac{\pi}{2}) \\ \sin(\theta(t) + \frac{\pi}{2}) \end{pmatrix}$$

$$= r \frac{d\theta}{dt} \vec{e}_\theta(t) \qquad \vec{e}_\theta \perp \vec{e}_r$$

$$a(t) = r \cdot \frac{d^2\theta}{dt^2} \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} + r \left(\frac{d\theta}{dt}\right)^2 \begin{pmatrix} -\cos\theta \\ -\sin\theta \end{pmatrix}$$

$$= r \frac{d^2\theta}{dt^2} \vec{e}_\theta(t) - r \left(\frac{d\theta}{dt}\right)^2 \vec{e}_r(t)$$

$$= \vec{a}_\theta(t) + \vec{a}_r(t)$$

Tangential- Zentripetal-

Beschleunigung