2.2.6 Leistung

Leistung $\equiv$ Arbeit in Zeitintervall

$$\langle P \rangle = \frac{A}{\Delta t}$$ Mittlere L.

$$P = \frac{dA}{dt}$$ Instantane L.

$$[P] = 1\,kg\,\frac{m^2}{s^3} = 1\,W$$

$$1\,Ps = 735\,W$$

Demo: Student läuft Treppe hoch

$$\langle P \rangle = \frac{m \cdot g \cdot h}{\Delta t} = \frac{65\,kg \cdot 9{,}8\,\frac{m}{s^2} \cdot 3{,}5\,m}{3\,s}$$

$$= 743\,W$$

$$= 1{,}0\ Ps$$

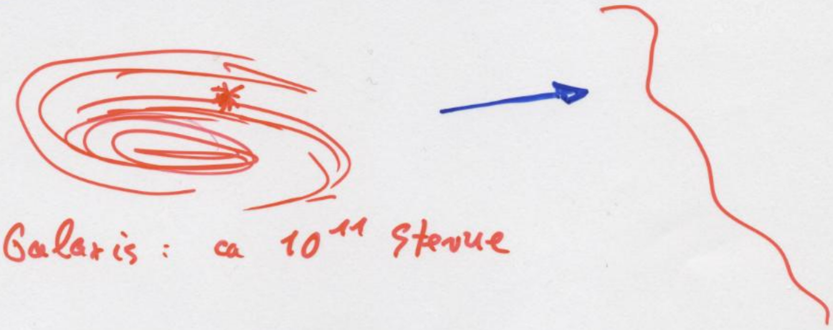
Mechanische / Elektr. Leistung

$$1\,W = 1\,N\,\frac{m}{s} = 1\,V \cdot A$$

Lampe: $\sim 50 - 100\ W$

2.3 Systeme von Massepunkten

2.3.1 Schwerpunkt und Impuls



Unsere Galaxis: ca 10^{11} Sterne

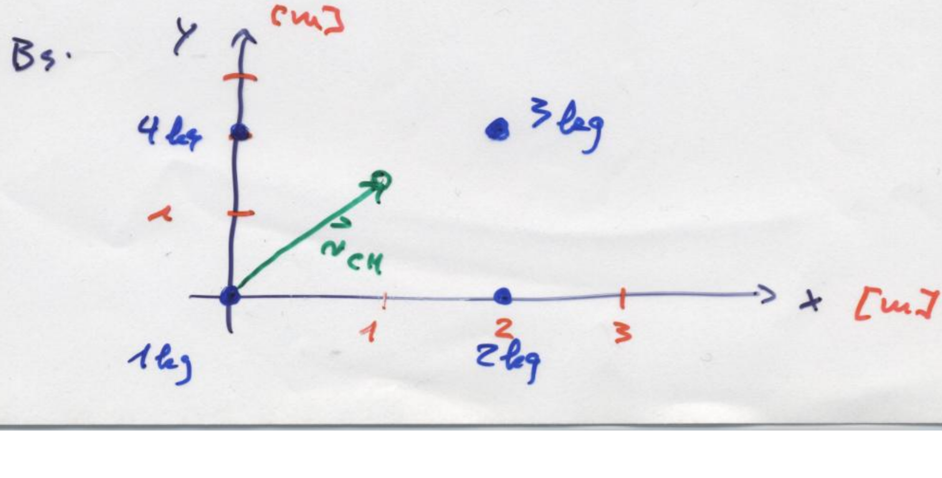
Schwerpunkt: [Center of Mass ≡ CM]

$$\vec{r}_{CM} \equiv \frac{1}{M} \int \vec{r}(m)\, dm$$ für ∞ viele Teilchen

$$= \frac{1}{M} \int \vec{r} \cdot \underset{\text{Dichte}}{\rho(\vec{r})}\, \underset{\text{Volumenelement}}{dV}$$

$$\vec{r}_{CM} \equiv \frac{\sum^{N} m_i \vec{r}_i}{\sum^{N} m_i}$$ für endlich viele Teilchen

Bs.



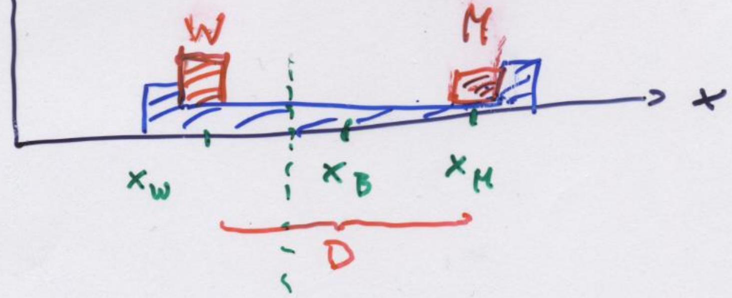
$$\vec{r}_{CM} = \left(1\,\text{kg} \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2\,\text{kg} \begin{pmatrix} 2\,\text{m} \\ 0 \end{pmatrix} + 3\,\text{kg} \begin{pmatrix} 2\,\text{m} \\ 2\,\text{m} \end{pmatrix} + 4\,\text{kg} \begin{pmatrix} 0 \\ 2\,\text{m} \end{pmatrix}\right) \cdot \frac{1}{(1+2+3+4)\,\text{kg}}$$

$$\underline{\vec{r}_{CM} = \begin{pmatrix} 1\,\text{m} \\ 1{,}4\,\text{m} \end{pmatrix}}$$

Anwendung: W + M im Boot.

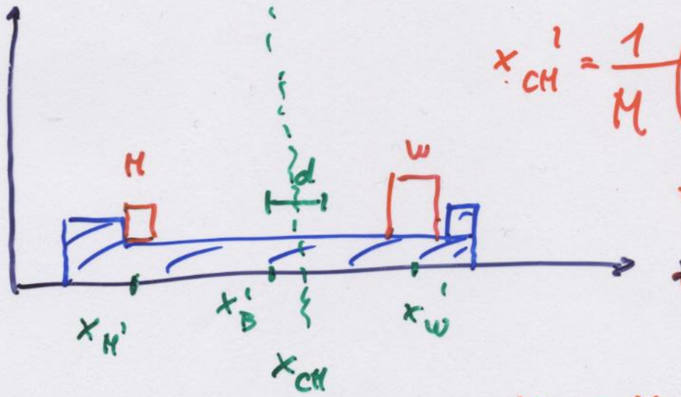
a

$$x_{CM} = \frac{1}{M}\left(m_W x_W + m_B x_B + m_M x_M\right)$$



b

$$x'_{CM} = \frac{1}{M}\Big((x_W - d)\, m_M + (x_B - d)\, m_B + (x_M - d)\, m_W\Big)$$



$$x_M - x_W = D$$

$$x_{CM} = x'_{CM}$$

$$\Rightarrow m_W \cdot x_W + \cancel{m_B \cdot x_B} + m_M \cdot x_M$$

$$= m_W x_M - m_W d + \cancel{m_B x_B} - m_B d + m_M x_W - m_M d$$

$$m_M = \frac{m_W (D-d) - m_B \cdot d}{D+d}$$

$m_W = 80\ kg$ $\quad d = 0{,}4\ m$

$m_B = 50\ kg$

$D = 3\ m$ $\quad \Rightarrow m_M = 57{,}6\ kg$

Bewegung von Systemen von Massep.

- $$\vec{V}_{CM} = \frac{d}{dt} \vec{r}_{CM} = \frac{\sum m_i \vec{v}_i}{M}$$

$$\Rightarrow M \cdot \vec{v}_{CM} \equiv \vec{P}_{CM} = \sum \vec{p}_i$$

Impuls

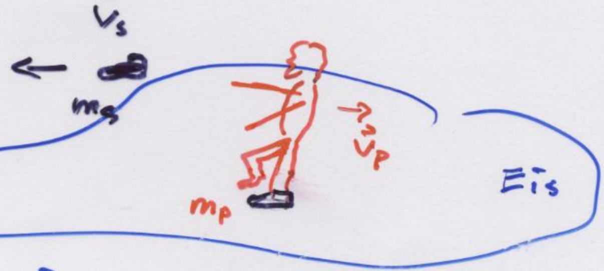
- $$\vec{a}_{CM} = \frac{\sum m_i \vec{a}_i}{M} ;$$

$$M \cdot \vec{a}_{CM} = \vec{F}_{CM} = \sum \vec{F}_i$$

Impulserhaltungssatz :

$$\boxed{\begin{array}{c} \sum \vec{F}_i = 0 \quad \hat{=} \sum \frac{d\vec{p}_i}{dt} = 0 \\ \Rightarrow \quad \vec{P}_{CM} = \text{const} \end{array}}$$

Beispiel a)



$$\left\{ \vec{P}_{CM} = \text{const} \right\} = \vec{p}_1 + \vec{p}_2$$

$$= m_s \cdot \vec{v}_s + m_p \cdot \vec{v}_p = 0$$

x-Richtung: $\hat{=} \; - m_s v_s + m_p v_p = 0$

$$\Rightarrow v_p = \frac{m_s}{m_p} v_s$$

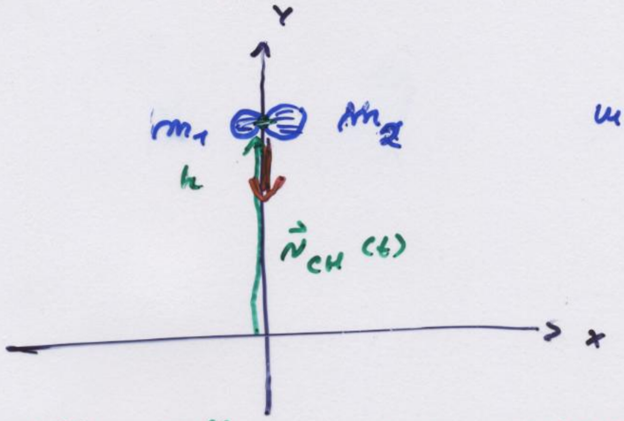
$m_p = 73\,\text{kg}$

$m_s = 2\,\text{kg}$ [Ski stiefel]

$v_s = 10\,\frac{m}{s}$

$$\Rightarrow v_p = 0{,}27\,\frac{m}{s}$$

Beispiel b) CM - Bewegung mit externer Kraft



$m_1 + m_2 = M$

1. Beide Bälle fallen, zusammengeklebt

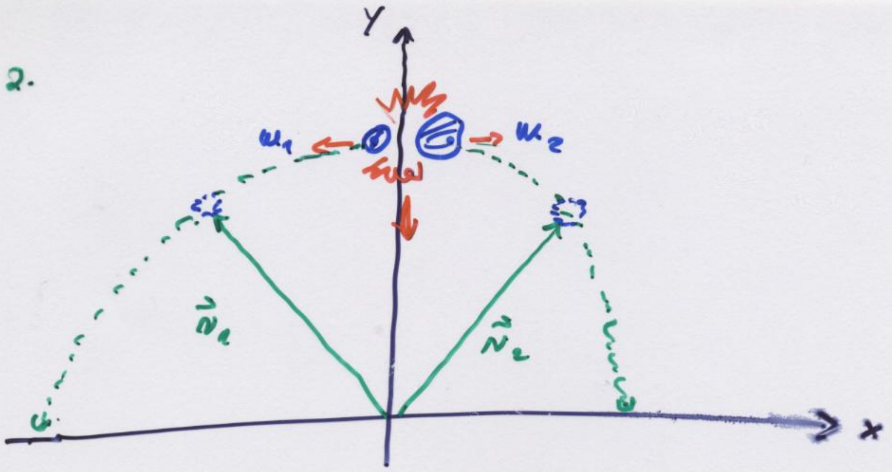
$$M = m_1 + m_2$$

$$\vec{F}_{CM} = m_1 \vec{g} + m_2 \vec{g} = M \cdot \vec{g} = \frac{d}{dt} \vec{P}_{CM}$$

$$\Rightarrow \vec{P}_{CM} = \begin{pmatrix} 0 \\ -gt \cdot M \end{pmatrix} ; \quad \vec{v}_{CM} = \begin{pmatrix} 0 \\ -gt \end{pmatrix}$$

$$\vec{r}_{CM} = \begin{pmatrix} 0 \\ h - \frac{g}{2} t^2 \end{pmatrix}$$

2.



$$\vec{r}_1(t) = \begin{pmatrix} -v_1 t \\ h - \frac{g}{2}t^2 \end{pmatrix} \qquad \vec{r}_2(t) = \begin{pmatrix} v_2 t \\ h - \frac{g}{2}t^2 \end{pmatrix}$$

$$\vec{r}_{CM}(t) = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M} = \frac{1}{M}\begin{pmatrix} -v_1 t\, m_1 + v_2 t\, m_2 \\ (h - \frac{g}{2}t^2)(m_1 + m_2) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ h - \frac{g}{2}t^2 \end{pmatrix}$$ wie bei den zusammengeklebten Bällen

$$\Rightarrow \vec{v}_{CM}(t) = \begin{pmatrix} 0 \\ -gt \end{pmatrix}$$

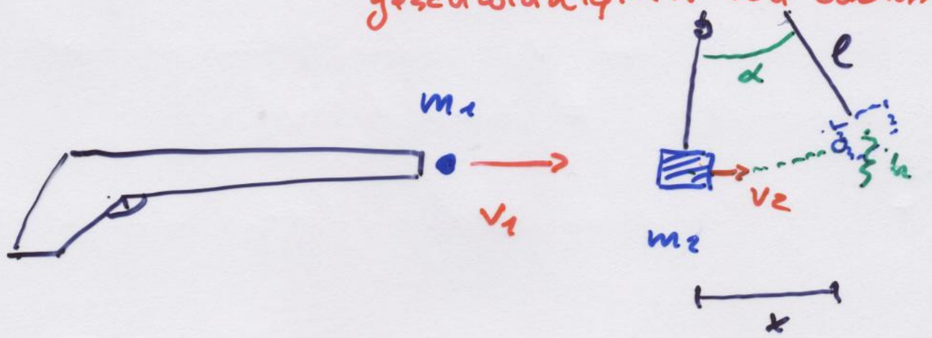
$$\vec{P}_{CM}(t) = M \cdot \vec{v}_{CM} = M \cdot \vec{g} \cdot t$$

$\neq$ const,

$$\text{da } \frac{d\vec{P}_{CM}(t)}{dt} = M \cdot \vec{g} \neq 0$$

Demo: Ballist. Pendel

Anwendung: z.B. Bestimmung der Mündungsgeschwindigkeit von Geschossen



$$m_1 v_1 = (m_1 + m_2)\, v_2$$

$$\frac{1}{2}(m_1 + m_2)\, v_2^2 = (m_1 + m_2)\, g h$$ Energie des Pendels

$$\Rightarrow v_2 \sqrt{2 \cdot g \cdot h}$$

$$= v_2 \sqrt{2 g \cdot \ell\,(1 - \cos\alpha)}$$

$$v_1 = \sqrt{2 g \ell\,(1 - \cos\alpha)}\;\frac{m_2 + m_1}{m_1}$$

$$\tan\alpha \approx \frac{x}{\ell}$$

$x = 8$ cm
$\ell = 88$ cm
$m_1 = 0{,}5$ g
$m_2 = 355$ g

$g = 9{,}8\ \frac{m}{s^2}$

$$\Rightarrow v_1 = 189\ \frac{m}{s}$$