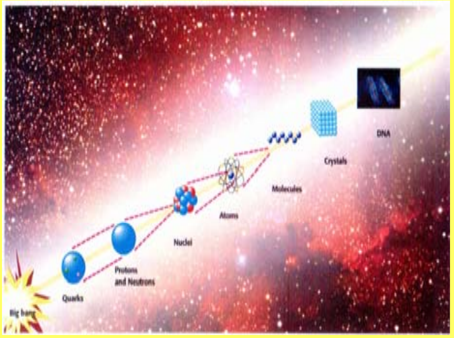
Basics of SUSY

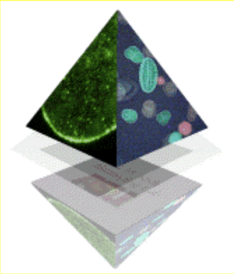
phenomenology

Dmitri Kazakov

JINR / ITEP



What is SUSY?



- **Supersymmetry** is a boson-fermion symmetry that is aimed to unify all forces in Nature including gravity within a singe framework

$$Q\,|\,boson> = |\,fermion> \qquad Q\,|\,fermion> = |\,boson>$$

$$[b,b]=0, \quad \{f,f\}=0 \;\Rightarrow \qquad \{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

- Modern views on supersymmetry in particle physics are based on string paradigm, though low energy manifestations of SUSY can be found (?) at modern colliders and in non-accelerator experiments

Motivation of SUSY in Particle Physics

- Unification with Gravity
- Unification of gauge couplings
- Solution of the hierarchy problem
- Dark matter in the Universe
- Superstrings

Unification with Gravity

$$spin\ 2\ \rightarrow\ spin\ 3/2\ \rightarrow\ spin\ 1\ \rightarrow\ spin\ 1/2\ \rightarrow\ spin\ 0$$

Unification of matter (fermions) with forces (bosons) naturally arises from an attempt to unify gravity with the other interactions

$$\{Q_\alpha^i, \bar{Q}_{\dot\beta}^j\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu \quad \Rightarrow \quad \{\delta_\varepsilon, \bar\delta_{\bar\varepsilon}\} = 2(\varepsilon\sigma^\mu\bar\varepsilon)P_\mu$$

$\varepsilon = \varepsilon(x)$ local coordinate transformation.

Local translation = general relativity !

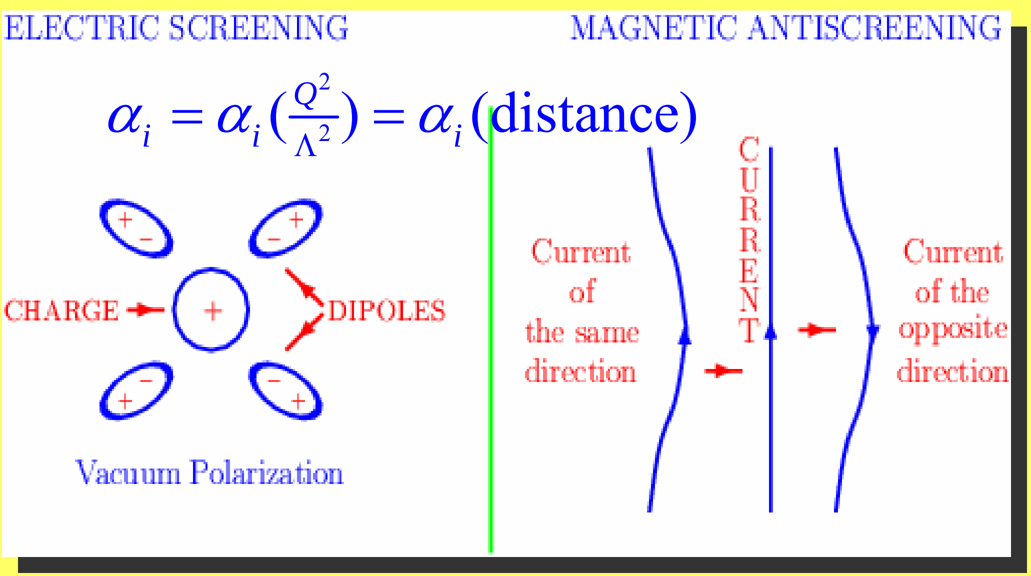
Supertranslation

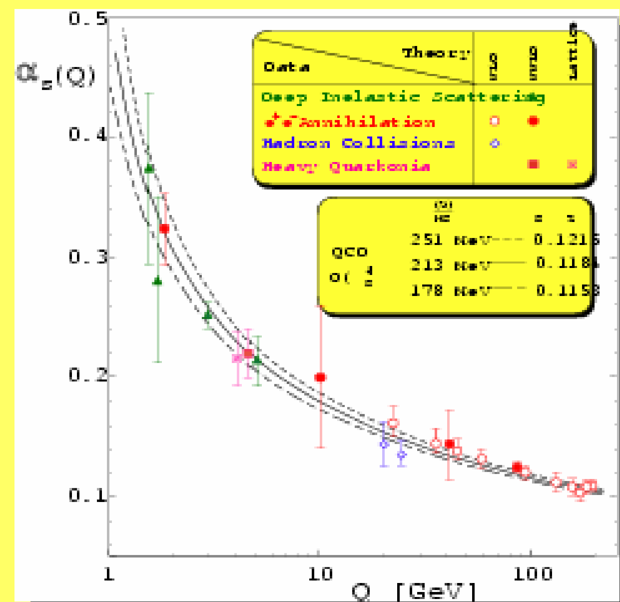
$$x_\mu \rightarrow x_\mu + i\theta\sigma_\mu\bar\xi - i\xi\sigma_\mu\bar\theta,$$
$$\theta \rightarrow \theta + \xi,$$
$$\bar\theta \rightarrow \bar\theta + \bar\xi$$

Motivation of SUSY in Particle Physics

- Unification of gauge couplings

	Low Energy		⇒	High Energy
$SU_c(3)$	$SU_L(2)$	$U_Y(1)$	⇒	G_{GUT} (or G^n + symm)
gluons	*W, Z*	*photon*	⇒	*gauge bosons*
quarks	*leptons*		⇒	*fermions*
g_3	g_2	g_1	⇒	g_{GUT}





Running of the strong coupling

Motivation of SUSY

RG Equations $\frac{d\tilde{\alpha}_i}{dt} = b_i\tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \alpha_i/4\pi = g_i^2/16\pi^2, \quad t=\log(Q^2/\mu^2)$

$$SM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam}\begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$MSSM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam}\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs}\begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

Input

$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$

$\sin^2\theta_{\overline{MS}} = 0.23146 \pm 0.00017$

$\alpha_s(M_Z) = 0.1184 \pm 0.0031$

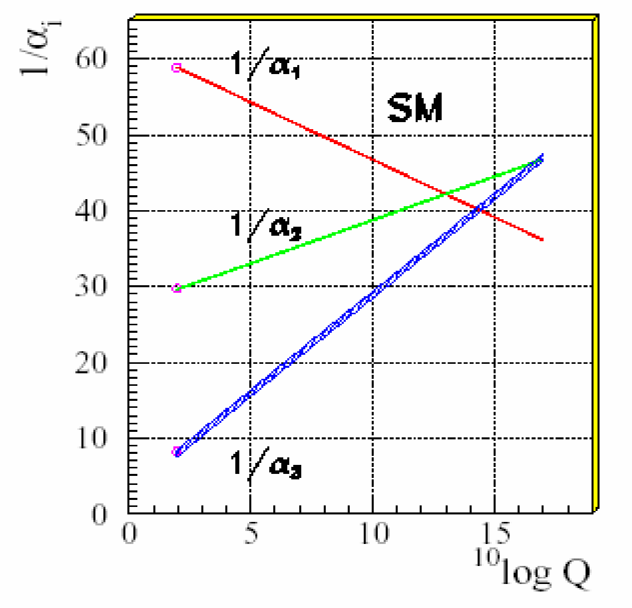
Output

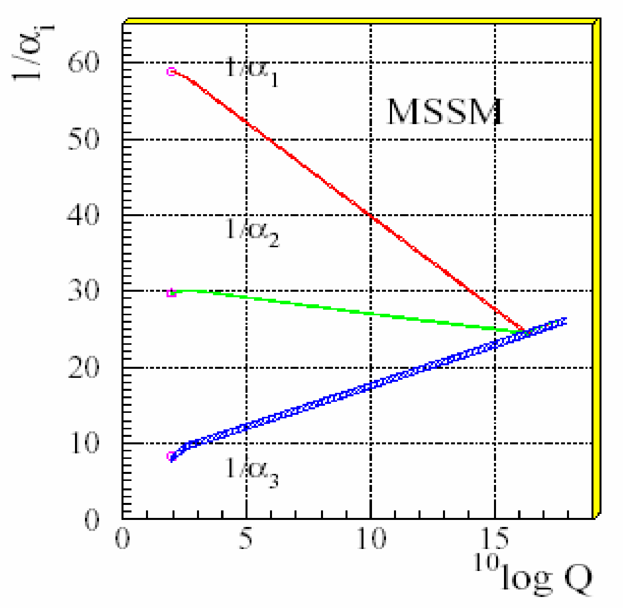
$M_{SUSY} = 10^{3.4\pm0.9\pm0.4}$ GeV

$M_{GUT} = 10^{15.8\pm0.3\pm0.1}$ GeV

$\alpha_{GUT}^{-1} = 26.3 \pm 1.9 \pm 1.0$

Unification of the Coupling Constants in the SM and in the MSSM





SUSY yields unification!

Motivation of SUSY

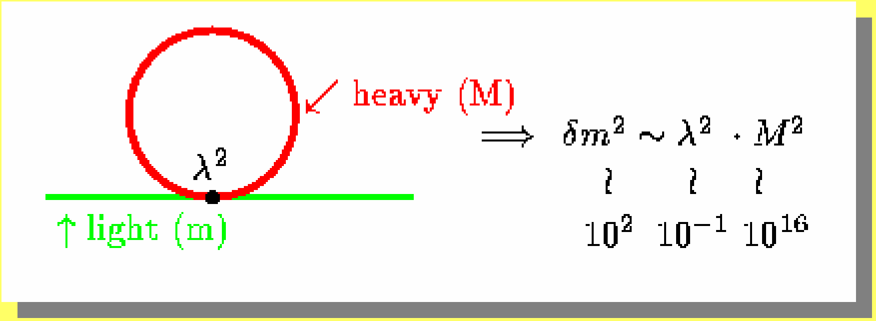
- Solution of the Hierarchy Problem

$$m_H \sim v \sim 10^2 \text{ GeV}$$

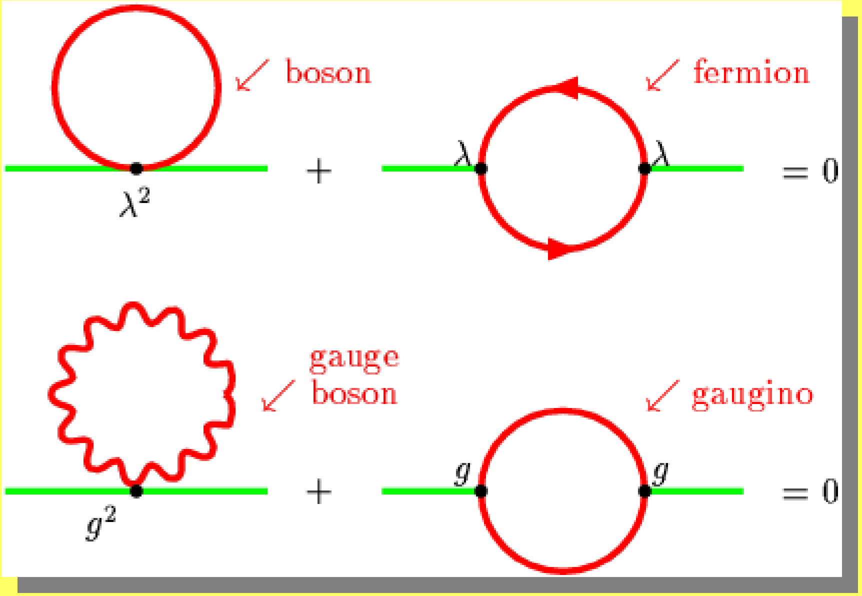
$$m_\Sigma \sim V \sim 10^{16} \text{ GeV}$$

$$\frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1$$

Destruction of the hierarchy by radiative corrections



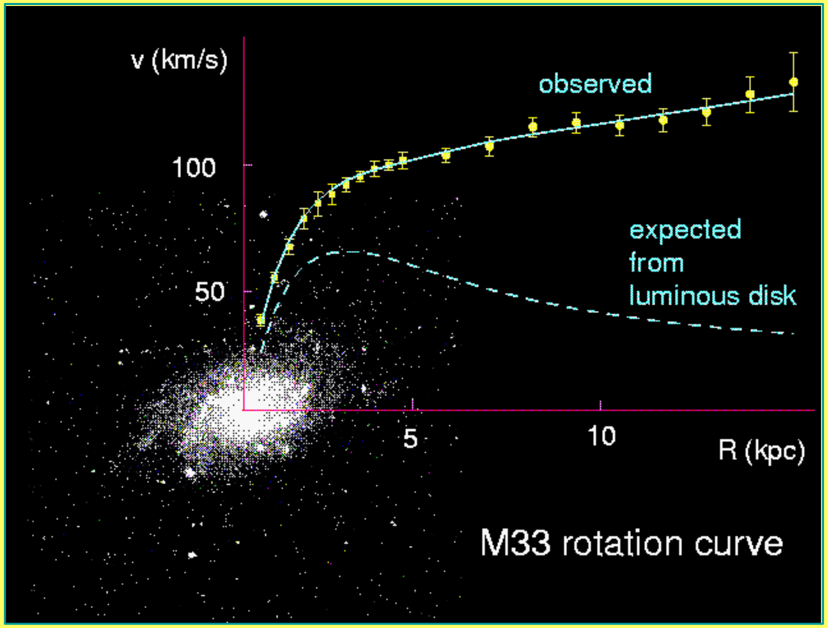
Cancellation of quadratic terms



$$\sum_{bosons} m^2 = \sum_{fermions} m^2$$

Motivation of SUSY

- Dark Matter in the Universe

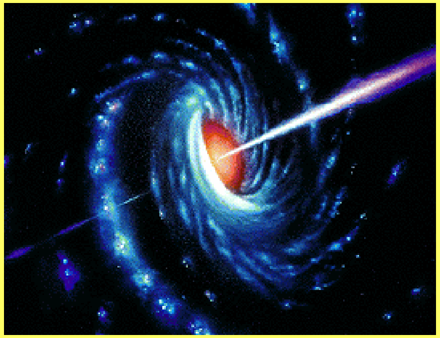


The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.

Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter

SUSY provides a candidate for the Dark matter – a stable neutral particle

Cosmological Constraints



New precise cosmological data

$$\Omega h^2 = 1 \quad \Longleftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%$$

$$\Omega_{DarkMatter} \approx 23 \pm 4\%$$

$$\Omega_{Baryon} \approx 4\%$$

- Supernova Ia explosion
- CMBR thermal fluctuations

Dark Matter in the Universe:

→ Hot DM (not favoured by galaxy formation)

→ Cold DM (rotation curves of Galaxies) ← SUSY

Superalgebra

(Super) Algebra

Lorentz Algebra

$$[P_\mu, P_\nu] = 0,\ [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),$$

SUSY Algebra

$$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,$$

$$[Q^i_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta,\ [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\tfrac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},$$

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

$$\alpha, \dot\alpha, \beta, \beta = 1,2;\ i, j = 1,2,\ldots,N.$$

Superspace

$$x_\mu \rightarrow x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha}$$

$\alpha, \dot\alpha = 1,2$

Grassmannian parameters

$$\vartheta^2_\alpha = 0,\ \bar\vartheta^2_{\dot\alpha} = 0$$

SUSY Generators

$$Q_\alpha = \frac{\partial}{\partial\vartheta_\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar\vartheta_{\dot\alpha}} + i\theta_\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$Q^2_\alpha = 0,\ \bar{Q}^2_{\dot\alpha} = 0$$

This is the only possible graded Lie algebra that mixes integer and half-integer spins and changes statistics

Quantum States

Quantum states: Vacuum = $|E,\lambda>$ $Q|E,\lambda>=0$

Energy helicity

$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0$

State	Expression	# of states
vacuum	$\|E,\lambda>$	1
1-particle	$\bar{Q}_i\|E,\lambda>=\|E,\lambda+1/2>$	$\binom{N}{1}=N$
2-particle	$\bar{Q}_i\bar{Q}_j\|E,\lambda>=\|E,\lambda+1>$	$\binom{N}{2}=\frac{N(N-1)}{2}$
…	…	…
N-particle	$\bar{Q}_1\bar{Q}_2...\bar{Q}_N\|E,\lambda>=\|E,\lambda+N/2>$	$\binom{N}{N}=1$

Total # of states: $\sum_{k=0}^{N}\binom{N}{k}=2^N=2^{N-1}\,bosons+2^{N-1}\,fermions$

SUSY Multiplets

Chiral multiplet $N = 1,\ \lambda = 0$

helicity	-1/2	0	1/2
# of states	1	2	1

(φ, ψ) — scalar, spinor

Vector multiplet $N = 1,\ \lambda = 1/2$

helicity	-1	-1/2	1/2	1
# of states	1	1	1	1

(λ, A_μ) — spinor, vector

Members of a supermultiplet are called **superpartners**

N=4	SUSY YM	helicity			-1	–1/2	0	1/2	1		
	λ = -1	# of states			1	4	6	4	1		
N=8	SUGRA	helicity	-2	–3/2	–1	–1/2	0	1/2	1	3/2	2
	λ = -2	# of states	1	8	28	56	70	56	28	8	1

$N \leq 4S$ ← spin

$N \leq 4$ For renormalizable theories (YM)

$N \leq 8$ For (super)gravity

Simplest (N=1) SUSY Multiplets

Bosons and Fermions come in pairs

$$(\varphi, \psi) \qquad (\lambda, A_\mu) \qquad (\tilde{g}, g)$$

Spin 0	Spin 1/2	Spin 1/2	Spin 1	Spin 3/2	Spin 2
scalar	chiral fermion	mayorana fermion	vector	gravitino	graviton

SUSY Transformation

N=1 SUSY Chiral supermultiplet:

parameter of SUSY transformation (spinor)

spin=0 → $\delta_\varepsilon A = \sqrt{2}\varepsilon\psi,$

spin=1/2 → $\delta_\varepsilon \psi = i\sqrt{2}\sigma^\mu \bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F,$

$\delta_\varepsilon F = i\sqrt{2}\bar{\varepsilon}\sigma^\mu \partial_\mu \psi$

Auxiliary field

(unphysical d.o.f. needed to close SYSY algebra)

SUSY multiplets ➡ Superfiled in Superspace

$(y = x + i\theta\sigma\bar{\theta})$

superfield

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

$$= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x)$$

$$+\sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)$$

Expansion over grassmannian parameter

component fields

$$\theta^2 = \theta_1\theta_2, \quad \theta_1^2 = \theta_2^2 = 0!$$

Gauge superfields

Gauge superfield

$$V(x,\theta,\bar\theta) = C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + i\theta\theta M(x) - i\bar\theta\bar\theta M^+(x)$$
$$-\theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta[\bar\lambda(x) + i\bar\sigma^\mu\partial_\mu\chi(x)] - i\bar\theta\bar\theta\theta[\lambda(x) + i\sigma^\mu\partial_\mu\bar\chi(x)]$$
$$+\tfrac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) + \tfrac{1}{2}\Box C(x)]$$

Field strength tensor

$$W_\alpha = -\tfrac{1}{4}\bar D^2 e^V D_\alpha e^{-V}$$

Gauge transformation

$$V \to V + \Phi + \bar\Phi$$

$$C \to C + A + A^*$$
$$\chi \to \chi - i\sqrt{2}\psi$$
$$M \to M - 2iF$$
$$v_\mu \to v_\mu - i\partial_\mu(A - A^*)$$
$$\lambda \to \lambda$$
$$D \to D$$

Wess-Zumino gauge

$$C = \chi = M = 0$$

physical fields

Covariant derivatives

$$D_\alpha = \tfrac{\partial}{\partial\vartheta_\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\vartheta^{\dot\alpha}\partial_\mu$$
$$\bar D_{\dot\alpha} = -\tfrac{\partial}{\partial\bar\theta_\alpha} - i\theta_{\dot\alpha}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \tfrac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma^\mu D_\mu\bar\lambda$$

How to write SUSY Lagrangians

1st step

Take your favorite Lagrangian written in terms of fields

2nd step

Replace $Field\ (\varphi, \psi, A_\mu) \Rightarrow Superfield\ (\Phi, V)$

3rd step

Replace

$$Action = \int d^4x\, L(x) \quad \Rightarrow \quad \int d^4x\, d^4\theta\, L(x, \theta, \bar{\theta})$$

Grassmannian integration in superspace $\int d\theta_\alpha = 0, \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$

SUSY Lagrangian (Matter)

Superfields

$$L = \int d^2\theta d^2\bar{\theta}\, \Phi_i^+ \Phi_i + \int d^2\theta\, (\lambda_i \Phi_i + \tfrac{1}{2} m_{ij} \Phi_i \Phi_j + \tfrac{1}{3} y_{ijk} \Phi_i \Phi_j \Phi_k) + h.c.]$$

Kinetic term

Superpotential

Components

$$L = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i + F_i^* F_i$$

no derivatives

$$+ [\lambda_i F_i + m_{ij}(A_i F_j - \tfrac{1}{2}\psi_i \psi_j) + y_{ijk}(A_i A_j F_k - \psi_i \psi_j A_k) + h.c.]$$

Constraint $\frac{\delta L}{\delta F_k} = F_k^* + \lambda_k + m_{ik} A_i + y_{ijk} A_i A_j = 0 \Rightarrow F_k$

$$L = i\partial_\mu \bar{\psi}_i \bar{\sigma}^\mu \psi_i + A_i^* \Box A_i - \tfrac{1}{2} m_{ij} \psi_i \psi_j - \tfrac{1}{2} m_{ij}^* \bar{\psi}_i \bar{\psi}_j$$

$$- y_{ijk} \psi_i \psi_j A_k - y_{ijk}^* \bar{\psi}_i \bar{\psi}_j A_k^* - V(A_i, A_j)$$

$$V = F_k^* F_k$$

SUSY Lagrangians (gauge)

Gauge fields

$$L = \frac{1}{4}\int d^2\theta \; W^{\alpha} W_{\alpha} + \int d^2\bar{\theta} \; \overline{W}^{\dot{\alpha}} \overline{W}_{\dot{\alpha}} = \frac{1}{2} D^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\lambda\sigma^{\mu} D_{\mu} \bar{\lambda}$$

Gauge transformation

$$\Phi \to e^{-ig\Lambda}\Phi, \; \Phi^+ \to \Phi^+ e^{ig\Lambda^+}, \; V \to V + i(\Lambda - \Lambda^+)$$

Gauge invariant interaction with matter (covariant derivative)

$$\Phi^+\Phi \;\to\; \Phi^+ e^{gV}\Phi$$

Gauge Invariant SUSY Lagrangian

Superfields

$$L_{SUSY\ YM} = \tfrac{1}{4}\int d^2\theta\ \mathrm{Tr}(W^\alpha W_\alpha) + \tfrac{1}{4}\int d^2\bar\theta\ \mathrm{Tr}(\overline{W}^\alpha \overline{W}_\alpha)$$
$$+\int d^2\theta d^2\bar\theta\ \bar\Phi_{ia}(e^{gV})^a_b\Phi^b_i + \int d^2\theta\ \mathcal{W}(\Phi_i) + \int d^2\bar\theta\ \overline{\mathcal{W}}(\bar\Phi_i)$$

Components

$$L_{SUSY\ YM} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^a\sigma^\mu D_\mu\bar\lambda^a + \tfrac{1}{2}\underline{D^a D^a}$$
$$+(\partial_\mu A_i - igv^a_\mu T^a A_i)^\dagger(\partial_\mu A_i - igv^a_\mu T^a A_i) - i\bar\psi_i\bar\sigma^\mu(\partial_\mu\psi_i - igv^a_\mu T^a\psi_i)$$
$$-\underline{D^a} gA_i^\dagger T^a A_i - i\sqrt{2}gA_i^\dagger T^a\lambda^a\psi_i + i\sqrt{2}g\bar\psi_i T^a\bar\lambda^a A_i + \underline{F_i^\dagger F_i}$$
$$+\frac{\partial\mathcal{W}}{\partial A_i}\underline{F_i} + \frac{\partial\overline{\mathcal{W}}}{\partial A_i^\dagger}\underline{F_i^\dagger} - \tfrac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j - \tfrac{1}{2}\frac{\partial^2\overline{\mathcal{W}}}{\partial A_i^\dagger\partial A_j^\dagger}\bar\psi_i\bar\psi_j$$

Potential

$$D^a = -gA_i^\dagger T^a A_i,\quad F_i = -\frac{\partial\mathcal{W}}{\partial A_i} \quad\rightarrow\quad \mathrm{V} = \tfrac{1}{2}D^a D^a + F_i^\dagger F_i$$

Spontaneous Breaking of SUSY

Energy $E = <0|H|0>$

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta} P_\mu$$

$$E = \frac{1}{4}\sum_{\alpha=1,2} <0|\{Q^i_\alpha, \bar{Q}^j_\alpha\}|0> = \frac{1}{4}\sum_\alpha |Q_\alpha|0>|^2 \geq 0$$

$E = <0|H|0> \neq 0$ if and only if $Q_\alpha|0> \neq 0$